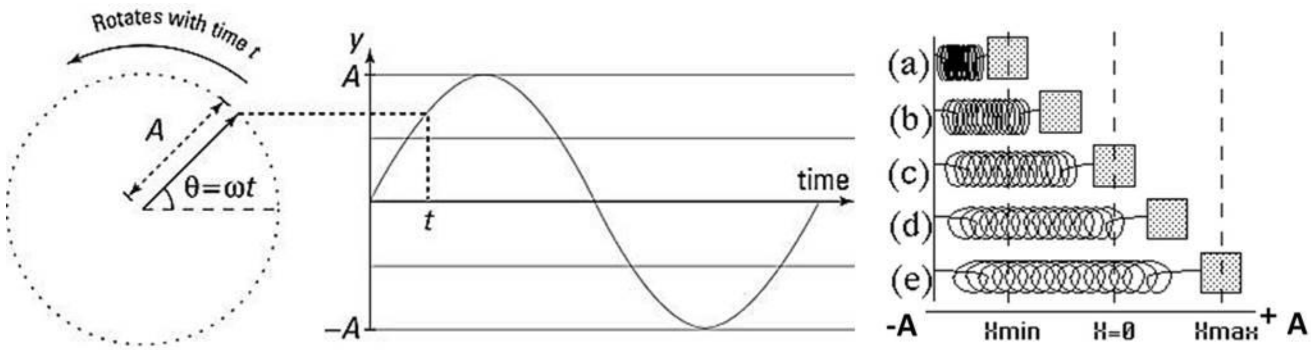
AP Physics 1- Simple Harmonic Motion and Waves Practice Problems

FACT: Simple harmonic motion (SHM) refers to the back-an-forth oscillation of an object, such as a mass on a spring and a pendulum. The position as a function of time graph is sinusoidal. SHM and uniform circular motion (UCM) are closely related, in fact, SHM describes the one dimensional motion of an object moving in UCM. Study the images below and recognize that the x-positon of an object in SHM is expressed as, x = Acosθ. Recall that θ = ωt and substitute this into the x-position expression to show that x = Acos ωt.



FACT: The amount of time per cycle is the period (T). The formula is $\frac{\#\ sec}{cycle}$. A cycle is considered a "round trip" of the oscillator. In physics, it is more accurate to record the time it takes for 10 cycles and divide, than to find the period for 1 cycle. The number of cycles completed in 1 second is the frequency (F). The formula is $\frac{\#\ cycles}{sec}$ and the unit of frequency in the hertz (Hz). The period and the frequency are inversely related.

Q1. A block oscillating on a spring moves from its position of max spring extension to max compression in 0.25 s. Determine the period and frequency of motion.

Q2. A student observing an oscillating block, counts 45.5 cycles of oscillation in one minute. Determine the frequency and period.

FACT: The frequency (f) and period (T) of a spring-block oscillator can actually be found using only the mass of the block and the force constant (k) of the spring. Frequency (f) = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and Period (T) = 2π $\sqrt{\frac{m}{k}}$ Recall the force of a spring is given by Hooke's Law; F_s = -kx, where k is the spring constant in N/m. Remember that the negative sign simply indicates that this is a "restoring" force. Also recall that the potential energy in a spring is U_s= ½ kx^2.

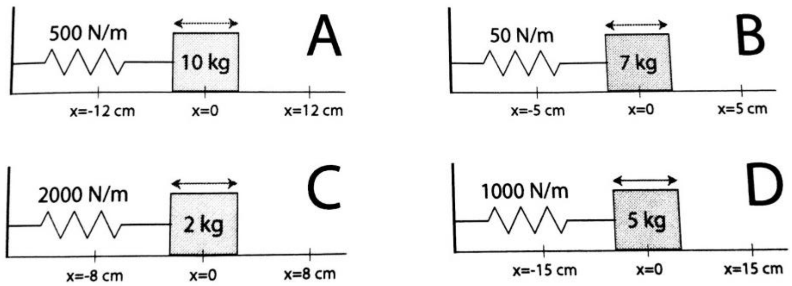
Q3. A block of mass m=2 kg is attached to a spring whose spring force constant is 300 N per meter. Calculate the frequency and period of the oscillations of this spring block system.

Q4. A block is attached to a spring and set into oscillatory motion and its frequency is measured. If this block were removed and replaced by a second block with 1/4 the mass of the first block, how would the frequency of the oscillations compare to that of the first block

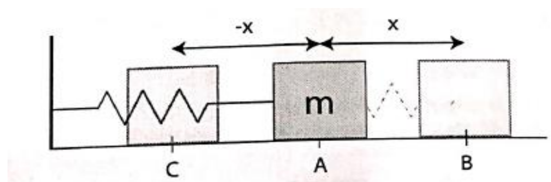
Fact: In simple harmonic motion both the frequency and the period are independent of the amplitude.

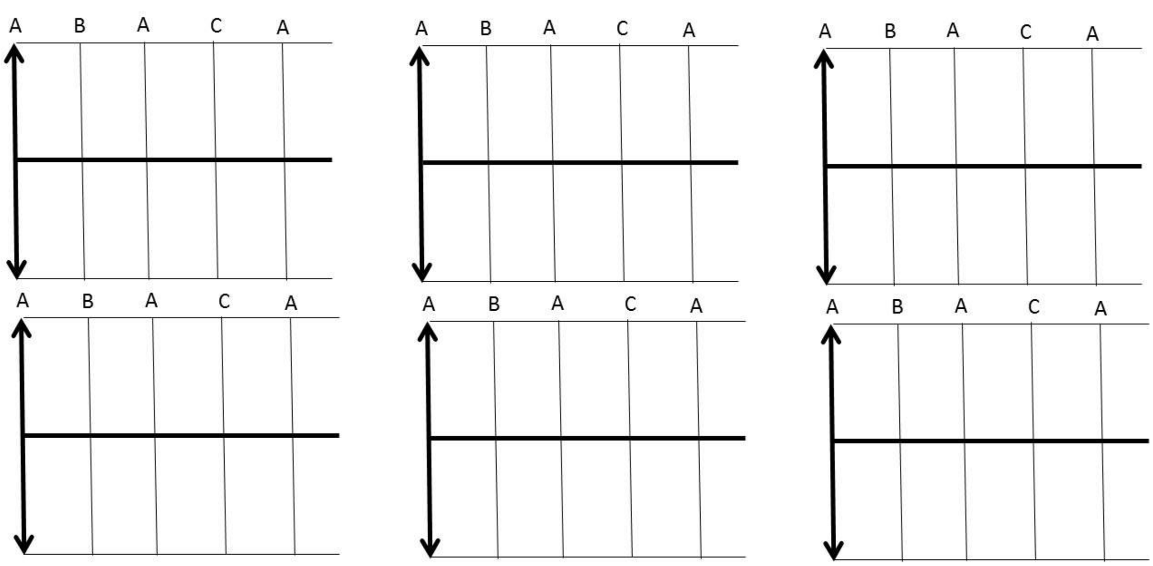
Q5. A student performs an experiment with a spring block simple harmonic oscillator. In the first trial the amplitude of the oscillations is 3 cm, while in the second trial using the same spring/block the amplitude of the oscillations is 6 cm. Compare the values of the period, frequency, and the maximum speed of the block between these two trials.

Q6. Rank the following horizontal spring-block oscillators resting on frictionless surfaces in terms of their period, from longest to shortest.



Q7. For the spring-block oscillator shown, sketch the graphs for x(t), v(t), PE(t), KE(t), F(t), and a(t). Label all axes.





Q8. A 2 kg block is attached to a spring. A force of 20 N stretches the spring to a displacement of 50 cm find: a. the spring constant, b. the total energy, c. the speed of the block at the equilibrium position, d. the speed of the block at X equals 30 cm, e. the speed of the block at X equals -40 cm, f. the acceleration at the equilibrium position, g. the magnitude of the acceleration at X equals 50 cm, h. the net force on the block at the equilibrium position, i. the net force at X equals 25 cm, and j. the position where the kinetic energy is equal to the potential energy.

Q9. A cart of mass 0.5 kg is attached to a spring of spring constant 30 N/m on a frictionless air track. The cart is stretched 10 cm from the equilibrium position and released from rest. Calculate the maximum speed of the cart.

Q10. Draw a FBD for a mass hanging vertically on a spring at its equilibrium position. Sum the forces for this scenario.

FACT: When spring-block system oscillates vertically, we need to account for the force of gravity. When the mass-spring system are not moving we can say ky = mg. Solving for y gives us our "new" equilibrium position; y = $\frac{mg}{k}$. You only need to worry about this if a question asks about the total length of a spring at a given moment. Otherwise, continue to use horizontal equations.

Q11. A block of mass, m=1.5 kg is attached to the end of a vertical spring (k=300 N/m). After the block comes to rest, it is pulled down a distance of 2.0 cm and released. A). Find the frequency of the oscillations B). What are the minimum and maximum amounts of stretch of the spring during the oscillations?

Q12. A 2-kg block attached to an un-stretched spring (k = 200 N/m) is released from rest. a). Determine the period of the blocks oscillation; b). What is the maximum displacement of the block from its equilibrium while undergoing SHM?

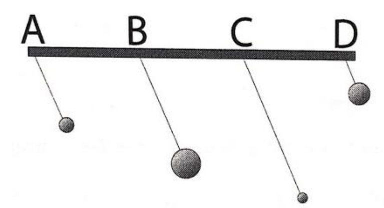
FACT: A simple (or ideal) pendulum is a weight of mass (m) attached to a string or massless rod (L) that swings, without friction, about the vertical equilibrium position. Simply pendulums also demonstrate SHM.

Q13. Sketch a pendulum of mass, m and length L, which is at angle, θ, from its equilibrium position. Draw the FBD for this pendulum. Based on your FBD, what is the restoring force for a pendulum in SHM?

FACT: The angular frequency of an ideal pendulum for small angles of theta (θ) is given by $\omega = \sqrt{\frac{g}{l}}$. We know the period to be $T_p = \frac{2\pi}{\omega}$ Therefore, substituting in the angular frequency gives us $T_p = 2\pi\sqrt{\frac{l}{g}}$.

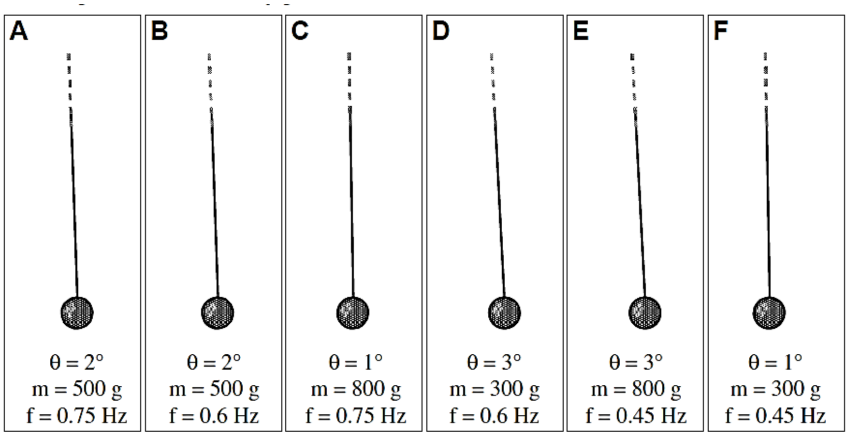
Q14. A simple pendulum has a period of one second on Earth. a). What would its period be on the moon where gravity is 6 times smaller than on Earth? b). Calculate a new length for the pendulum that would compensate for the difference in gravity and return the period to one second.

Q15. A grandfather clock is designed such that each swing of the pendulum takes one second. How long is the pendulum of a grandfather clock?



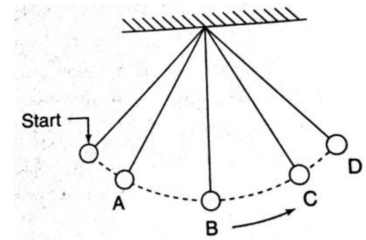
Q16. Rank the following pendulums of uniform mass density from highest to lowest frequency.

Q17. The six figures below show metal spheres hung on the ends of strings. The spheres have been pulled to the side and released so that they are swinging back and forth. For each sphere-string system the diagrams give the mass of the sphere, the frequency of the swing, and how far, in terms of the angle from the vertical, that the spheres were initially pulled to the side. Rank these systems on the basis of the length of the string.



Q18. The period of an ideal pendulum is T. If the mass of the pendulum is tripled while its length is quadrupled, what is the new period of the pendulum expressed in terms of T?

FACT: Recall from previous units that we can use the geometry of a pendulum to determine the height = L (1-cosθ). We can then analyze the pendulum's SHM in terms of its kinetic and potential energy.

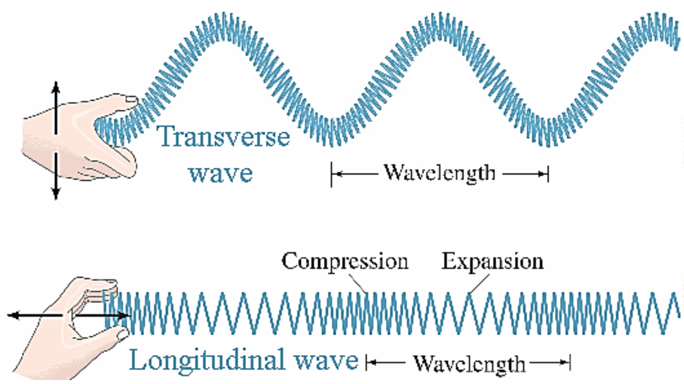


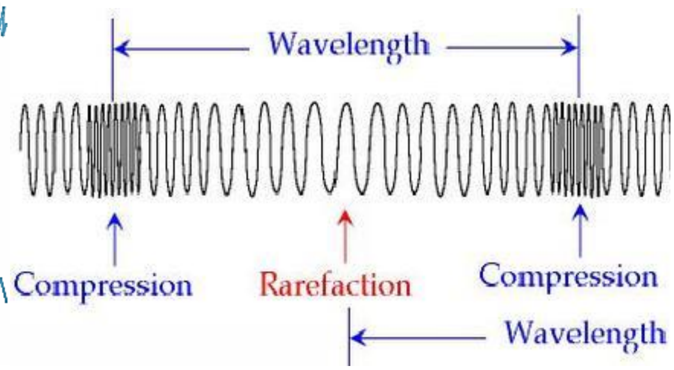
Q19. A pendulum of length 20 cm and mass 1 kg is displaced an angle of 10° from the vertical. What is the maximum speed pendulum?

Q20. A pendulum of length 0.5 m and mass 5 kg is displaced an angle of 14° from the vertical. What is the speed of the pendulum when its angle from the vertical is 7°?

Q21. A pendulum is released from rest at the start position on the diagram shown and oscillates in SHM through positions A-D. The gravitational field strength on the surface of Jupiter is 26 N/kg and on the Moon, 1.6 N/kg. Rank the pendulum's period on these two planets and Earth.

FACT: A wave is a repeated disturbance which carries energy through a medium. A transverse wave travels (propagates) in a direction perpendicular to the direction in which the medium is vibrating. Basically, the wave oscillates perpendicular to its direction of travel. When a medium vibrates parallel to the direction of the wave, the wave is longitudinal (sound). Please understand that a single point on the medium is only moving vertically (up and down) for a transverse wave while the disturbance (i.e., energy) moves left to right. For a longitudinal wave, a single point moves to the right and back to the left, while the energy travels from the left side to the right side.





FACT: Important characteristics of a traveling wave include its amplitude, period, frequency, wavelength, and velocity. The equation v = λf shows how the wave speed, wavelength, and frequency are interconnected.

FACT: The energy carried by a wave depends on the wave's amplitude. When a wave changes materials, its frequency stays the same. From a semiquantitative perspective (v = λf), if the velocity increases as a wave enters a new medium, then the wavelength must also increase, since the frequency never changes.

Q22. A wave traveling on a rope has a frequency of 2.5 Hz. If the speed of the wave is 1.5 m/s, what are its period and wavelength?

Q23. The period of a traveling wave is 0.5 seconds, its amplitude is 10 cm, and its wavelength is 0.4 m. What are its frequency and wave speed?

FACT: We can derive any equation for the speed of a transverse wave on a stretched string. The equation is v = $\sqrt{\frac{Ft}{\mu}}$. In this equation μ is the linear mass density and the units are mass/Length, or kg/m, μ = $\frac{m}{L}$. This equation shows that velocity does *not* depend on frequency or wavelength. So, because velocity is equal to frequency multiplied by lambda (v = λf) for a stretched string, varying the frequency will create different waves that have different wavelengths, but velocity will not vary. Again, this is for waves on a stretched string.

Wave rule #1: The speed of a wave is determined by the type of wave and the characteristics of the medium, not by the frequency.

Wave rule #2: When a wave passes into another medium its speed changes, but its frequency does not.

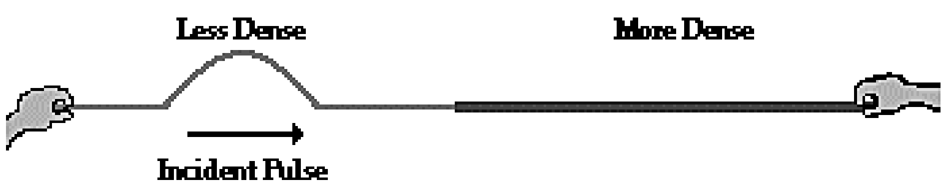
Q24. A horizontal rope with a linear mass density equal to 0.5 kg/meter sustains a tension of 60 N. The nonattached end is oscillated vertically with the frequency of 4 Hz. (a). What are the speed and wavelength of the resulting wave? (b.) How would you answer these questions if the frequency were increased to 5 Hz?

FACT: When a wave or pulse encounters a new medium or an obstacle, the interface of the two media is called the boundary and the behavior of a wave or pulse at that boundary is known as its boundary behavior. The original wave or pulse is often referred to as the incident wave or pulse.

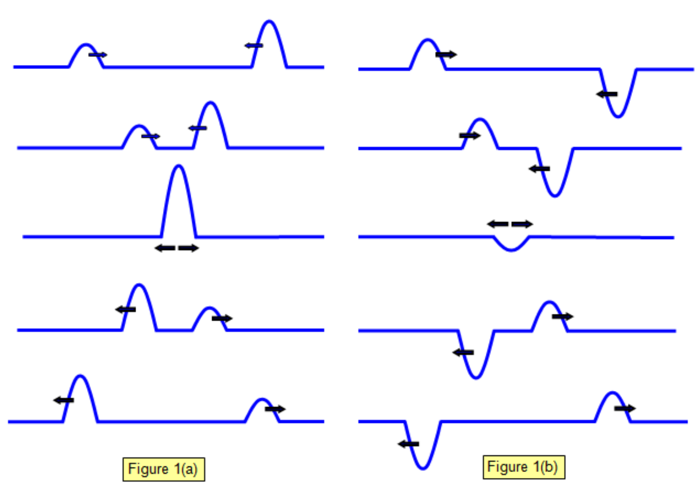
FACT: When a wave or pulse encounters a fixed end, a portion of the energy carried by the wave or pulse is reflected and inverted. The reflected wave or pulse has less energy (smaller amplitude), but the same speed and wavelength since the reflected wave is in the same medium. Note that if the end is free to the move (i.e., not fixed), the wave will reflect as described above, but it will NOT invert.

FACT: When a wave or pulse encounters a new medium and crosses the boundary, the behavior is dependent on the relative linear mass densities. If entering a denser medium, the reflected wave or pulse has a smaller amplitude, but the same speed and wavelength. However, the transmitted wave or pulse has a decreased velocity and thus a decreased wavelength. If entering a less dense medium, the reflected wave or pulse has a smaller amplitude, but the same speed and wavelength. However, the transmitted wave or pulse has an increased velocity and thus an increased wavelength.

Q25. Two ropes of unequal linear densities are connected, and a pulse is created in the rope on the left, which propagates to the right, toward the interface with the heavier rope. How do the speed and wavelength of the incident pulse compare to the speed and wavelength of the transmitted pulse? How do the speed and wavelength of the reflected pulse compare to the incident pulse?



FACT: Superposition is when two waves collide and interfere with each other forming a single wave and then continuing on their original direction. In constructive interference the crest of one wave overlaps the crest of another. The result is a wave of increased amplitude. In destructive interference the crest of one wave overlaps the trough of another. The result is a wave of reduced amplitude. Please see figures 1(a) and 1(b) to the right. I strongly encourage you to explore the following interactive: https://www.geogebra.org/m/BJ65gzqx



Q26. Two waves, one with an amplitude of 8 cm and the other with an amplitude of 3 cm, travel in the same direction on a single string and overlap. What are the maximum and minimum amplitudes of the string while these waves overlap?

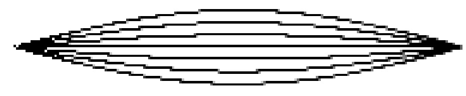
Q27. Please complete the "wave superposition supplement" handed out in class (or click here).

FACT: A standing wave occurs when waves with the same frequency and amplitude traveling in opposite directions meet. The waves appear to stay in one place. This is because of interference in which certain points (nodes) appear to be standing still and other points (antinodes) vibrate with maximum amplitude above and below the axis.

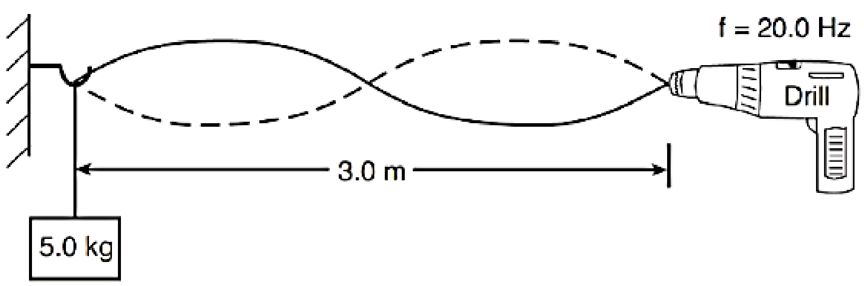
FACT: Nodes are areas of complete destructive interference, while antinodes are areas of complete constructive interference. Other points of a standing wave have amplitudes between these two extremes.

FACT: The wavelength (λ) of a standing wave is twice the node-to-node distance. So the distance between two successive nodes is equal to 1/2λ.

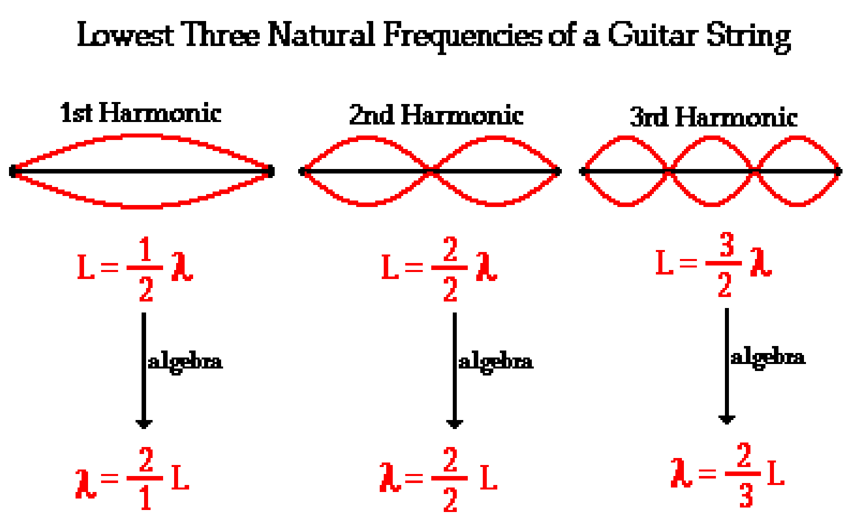
Q28. A guitar string of length 1 m is plucked creating one standing wave as shown to the right. What is the wavelength?



Q29. One end of a rope is attached to a variable speed drill and the other end is attached to a 5-kg mass. The rope is draped over a hook on a wall opposite the drill. When the drill rotates at a frequency of 20.0 Hz, standing waves of the same frequency are set up in the rope. (a). Determine the wavelength of the waves producing the standing wave pattern; (b). Calculate the speed of the wave on the rope.



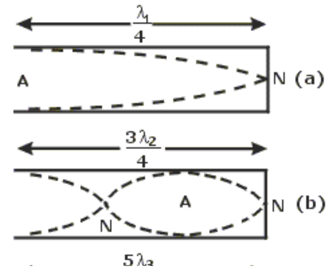
FACT: The fundamental frequency is created by holding a string at both ends, with half the wavelength set up between the nodes. This frequency is the first harmonic (n = 1; see diagram right). Let's derive some expressions to relate the harmonic (n), length of string (L), wavelength (λ), and frequency (f). First, the length of the string is half the harmonic number multiplied by the wavelength: $L = \frac{n\lambda}{2} \rightarrow \lambda = \frac{2L}{n}$. Next, let's substitute the wave equation ($v = f\lambda$ or $\lambda = \frac{v}{f}$) into the expression to get: $\frac{2L}{n} = \frac{v}{f} \rightarrow f = \frac{nv}{2L}$. Please note this is *all* also true for a pipe open at both ends.



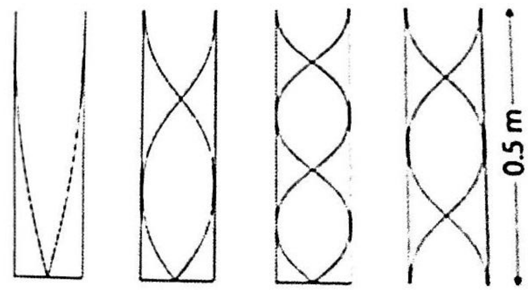
Q30. A guitar designer is designing an instrument in which the speed of the waves on the string should be 450 m/s. How long must the string be to produce a first fundamental frequency F# note at 370 Hz?

Q31. A string of length 12 m that is fixed at both ends supports a standing wave with a total of 5 nodes. What are the harmonic number and wavelength of this standing wave?

Q32. A string of length 10 m and mass 300 g is fixed at both ends, and the tension in the string is 40 N. What is the frequency of the standing wave for which the distance between the node and the closest antinode is 1 m?



FACT: When a pipe is closed at one end and open on the other, or a string is only attached to one end and free at the other end, the standing wave in this pipe must have a node at the closed end and an antinode at the open end. The frequency can be expressed as: f = $\frac{nv}{4L}$, with the speed (pipe) being speed of sound at ~340 m/s and the harmonic number (n) must be an odd integer. The wavelength is now λ = $\frac{4L}{n}$.



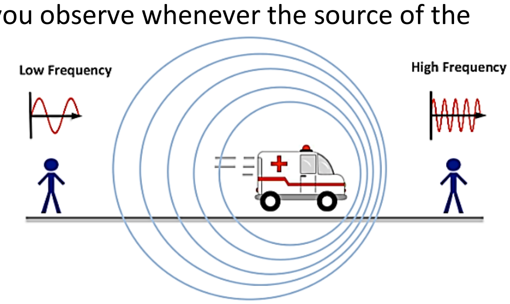
Q33. The diagram to the right shows air displacement of four standing waves inside a set of organ pipes. Assume the velocity of sound in air is 343 m/s. (a). What is the highest frequency for the waves shown? (b). what is the lowest frequency for the waves shown? (c). what is the longest wavelength shown in the pipes? (d). what is the shortest wavelength shown in the pipes?

Q34. A musician is designing a custom instrument which utilizes a tube open at both ends. Given the speed of sound in air is 343 m/s, how long should the musician make the tube to create an A (440 Hz) as the instrument's fundamental frequency?

FACT: Beats are rhythmic modulations in amplitude that occur when two sound waves whose frequencies are close, but not identical, interfere with one another. The result is a "wa-wa-wa" sound. As the waves travel they fluctuate between being in and out of phase. In terms of superposition, the waves repeatedly interfere constructively and then destructively. The number of beats per second is called the beat frequency and is equal to the difference between the frequencies of the two combining waves: $f_{beat} = |f1 - f2|$.

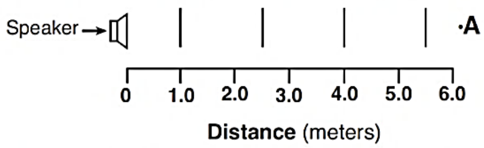
Q35. A piano tuner uses a tuning fork to adjust the key that plays the A note above middle C (f = 440 Hz). The tuning fork emits a perfect 440 Hz tone. When the tuning fork and the piano key are struck, beats of frequency 3 Hz are heard. (a). What is the frequency of the piano key? (b). If it is known that the piano key's frequency is too high, should the piano tuner tighten or loosen the wire inside the piano in order to tune it?

FACT: The Doppler Effect is the apparent change in a wave's frequency that you observe whenever the source of the wave is moving toward or away from you. When the source and observer are moving toward each other the observer perceives a shift to a higher frequency and vice versa. For the AP Exam, remember the effect is small, so at normal speeds a 200 Hz frequency may only change by tens of Hz and not hundreds of Hz.

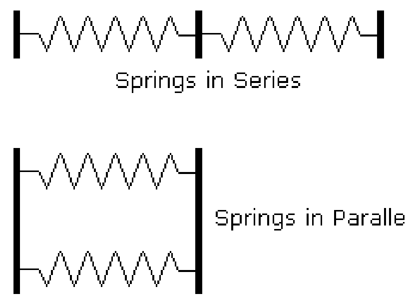


Q36. A car's horn is producing a sound wave having a constant frequency of 350 Hz. If the car moves toward a stationary observer at a constant speed, the frequency of the car's horn detected by this observer may be (circle one): 320 Hz – 330 Hz – 350 Hz – 380 Hz

Q37. The vertical lines in the diagram represent compressions in a sound wave of constant frequency propagating to the right from a speaker toward an observer at point A. (a). Determine the wavelength of this sound wave (b). The speaker is then moved at a constant speed toward the observer at A. Compare the wavelength of the sound wave received by the observer while the speaker is moving to the wavelength when the speaker was at rest.



FACT: Although the AP Exam does not cover the topic of spring combinations, it will help to conceptualize SHM. For springs in parallel, calculate an equivalent spring constant for the system by starting with Hooke's law, recognizing that displacement is the same for both springs. For springs in series, you will again calculate an equivalent spring constant for the system, beginning by realizing the force on each spring is the same according to Newton's third law of motion.



FACT: Damping is an influence upon an oscillating system that has the effect of reducing its oscillations through energy dissipated through nonconservative forces (i.e., friction)

Q38. Go back to Q7. and examine your sketched graphs. Now, consider a spring-block system in which there is damping. Sketch a graph of potential energy, kinetic energy, and amplitude as a function of time.

Q39. Rank the following spring-block oscillators in the diagram below from highest to lowest in terms of: 1). Equivalent spring constant; 2). Period of oscillation