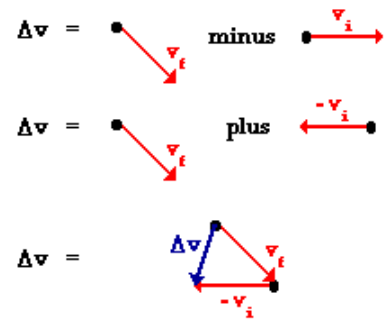
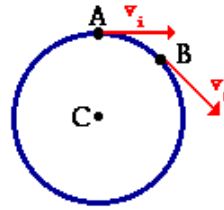


## AP Physics 1- Circular Motion and Rotation Practice Problems

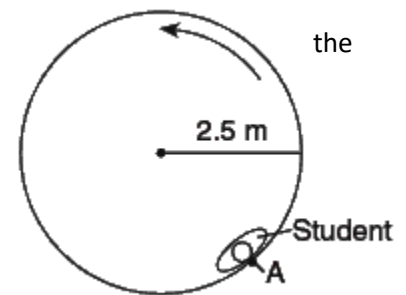
**FACT:** The motion of an object in a circular path at a constant speed is known as uniform circular motion (UCM). Because an object in UCM is constantly changing direction, it is also constantly changing its velocity. This means that an object in UCM is constantly accelerating. This is called centripetal acceleration ( $a_c$ ).



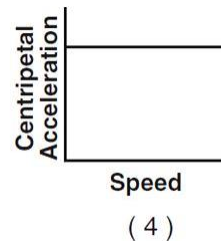
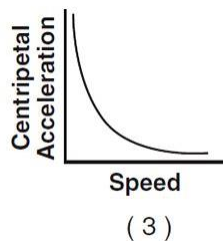
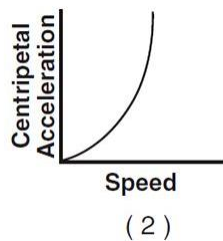
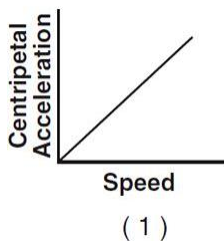
**FACT:** Recall, that  $\vec{a} = \Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ , divided by the change in time. We can show this vectorially in the diagrams to the right. Recall, that when we subtract vectors, we are really adding a negative vector. You can repeat this process around a circle and the acceleration vector will always point toward the center of the circle. Centripetal means “center-seeking”.

**FACT:** Now that we know the direction of the object’s acceleration is towards the center, we need to find the magnitude. The formula for the object’s centripetal acceleration is:  $a_c = \frac{v^2}{r}$ . The magnitude of centripetal acceleration clearly depends on the objects velocity ( $v$ ) and the radius ( $r$ ) of the circular path. The visual intuition for this idea is described in podcast #2 and it is important to acknowledge that differential calculus is needed for the derivation.

**Q1.** The diagram shows the top view of a 65 kg student at a point A on an amusement park ride. The ride spins the student in a horizontal circle of radius 2.5 m, at a constant speed of 8.6 m/s. The floor is lowered and the student remains against the wall without falling to the floor. (a) draw the vector for the centripetal acceleration on the diagram (b) Which direction is the force normal? (c) Which direction is force of friction on the student’s body?



**Q2.** Which graph best represents the relationship between the magnitude of the centripetal acceleration and the speed of an object moving in a circle of constant radius?

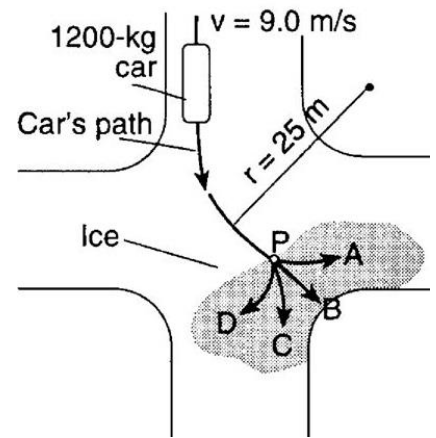


**Q3.** A 0.50 kg object moves in a horizontal circular path with a radius of 0.25 m at a constant speed of 4.0 m/s. What is the magnitude of the object’s acceleration?

**FACT:** Since an object traveling in a circular path has an inward acceleration, Newton’s 2<sup>nd</sup> Law states there must also be an inward net force. This centripetal force can be caused by a gravitational force, frictional force, tension force, or force applied. Now go back to Q1 and identify the centripetal force ( $F_N$ ). Let’s look at the expression for the centripetal force using  $F_c = ma_c$  and substituting in the formula for  $a_c$  we now get  $F_c = m \frac{v^2}{r}$ . Please note that it is incorrect to label the centripetal force “ $F_c$ ” on a FBD. You must identify the force actually causing the  $a_c$  and use that label in your FBD (e.g.,  $F_g$ ,  $mg$ ,  $F_T$ ,  $F_a$ ,  $F_f$ ). Always call the inward forces positive (+) when summing the forces.

**Q4.** An object of mass 5 kg moves at a constant speed of 6 m/s in a circular path of radius 2 m. Find the object’s acceleration and the net force responsible for the motion.

- Q5.** A 10.0 kg mass is attached to a string that has a breaking strength of 200N. If the mass is whirled in a horizontal circle of radius 80 cm, what maximum speed can it have?
- Q6.** An athlete who weighs 800 N is running around a curve at a speed of 5.0 m/s in an arc whose radius of curvature,  $r$ , is 5.0 m. Find the centripetal force acting on him. What provides the centripetal force? What could happen to him if  $r$  were smaller?
- Q7.** A 1200-kg car traveling at a constant speed of 9 m/s turns at an intersection. The car follows a horizontal circular path with a radius of 25 m to a point, P. At point P, the car hits an area of ice and loses all frictional force on its tires. (a) Which path does the car follow on the ice? (b) What was the magnitude of the centripetal force the instant before the car hit point P? (c) What is the coefficient of static friction for the car's tires on the pavement before hitting point P?
- Q8.** The maximum speed with which a 945-kg car makes a 180-degree turn is 10.0 m/s. The radius of the circle through which the car is turning is 25.0m. Calculate the force of friction and the coefficient of friction acting upon the car.



**FACT:** For objects moving in circular paths, you can characterize their motion around the circle in terms of frequency ( $f$ ) and period ( $T$ ). The frequency is the number of revolutions the object makes in a second ( $1/\text{s}$ , Hz). The period is the time it takes to complete one revolution. The period and frequency are expressed as:  $f = \frac{1}{T}$  and  $T = \frac{1}{f}$ .

- Q9.** Sophia makes 38 complete revolutions on the playground merry-go-round in 30 seconds. If the radius of the merry-go-round is 1 meter and her mass is 16 kg, determine the a). period of motion; b). frequency of motion; c). speed at which she revolves; d). centripetal force

**FACT:** Thus far, we have only explored centripetal motion in the context of horizontal circular paths. When objects are manipulated into vertical circular paths, we need to account for the force of gravity, or weight ( $mg$ ) of the object during certain points along the circle. Always, draw a FBD and sum the forces to avoid careless mistakes.

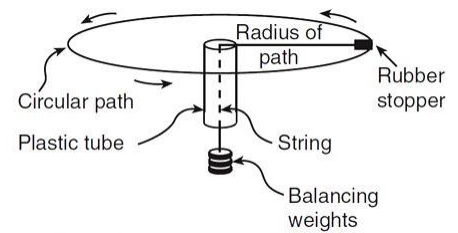
- Q10.** A rollercoaster car enters the circular-loop portion of the ride. At the very top of the circle, the speed of the car is 15 m/s, and the acceleration points downward. If the diameter of the loop is 40 m and the total mass of the car is 1200 kg, find the magnitude of the normal force exerted by the track on the car. Would the person in the car feel lighter or heavier?
- Q11.** Referring to Q10, how would the normal force change if the car was at the bottom of the loop? Please explain using qualitative and quantitative reasoning.

**FACT:** Many students are curious about the concept of the  $g$ -force felt by a person. Although, this concept is probably not on the AP exam, it is still interesting to consider. In the preceding question, you simplified the expression to  $F_N = m(\frac{v^2}{r} + g)$ . In parenthesis you have the acceleration due to gravity,  $g$ , and an additional term. It is this term ( $a_c$ ) that is the additional  $g$ -force felt by a person. If that term equals  $9.8 \text{ m/s}^2$ , then the person experiences two  $g$ 's. Now go back to Q10 and determine how many  $g$ 's are the people in the rollercoaster car experience at the bottom of the loop?

- Q12.** Sophia experiences a downward acceleration of  $15.6 \text{ m/s}^2$  at the top of a roller coaster loop and an upward acceleration of  $26.3 \text{ m/s}^2$  at the bottom of the loop. Use Newton's second law to determine the normal force acting upon Sophia's 864 kg roller coaster car.

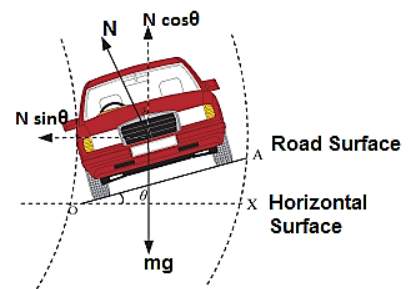
**Q13.** Sophia is riding on a roller coaster. Sophia is moving at 18.9 m/s over the top of a hill that has a radius of curvature of 12.7 m. Use Newton's second law to determine the magnitude of the applied force of the track pulling down upon Sophia's 621 kg roller coaster car.

**Q14.** In an experiment, a rubber stopper is attached to one end of a string that is passed through a plastic tube before weights are attached to the other end. The stopper is whirled in a horizontal circular path at a constant speed. (a) Describe what would happen to the radius of the circle if the student whirls the stopper at a greater speed without changing the balancing weights. (b) The rubber stopper is now whirled in a vertical circle at the same speed. Draw a FBD for this scenario.



**Q15.** Amedeo drives his car clockwise around a circular track with a radius of 30 meters. He completes 10 laps around the track in 2 minutes. a). Find Amedeo's total distance traveled, average speed, and centripetal acceleration. b). What is his displacement and average velocity?

**Q16.** Banking allows for cars to travel around a curve at or below a posted speed limit. In the diagram, the radius of curvature is 60 meters and the curve is banked at  $\theta = 11.8$  degrees. What recommended speed limit should be posted?



**FACT:** So far in this course we have only considered translational motion. Translational motion of a body can be either rectilinear or curvilinear, as we have studied in previous units, including an object manipulated into a circular path. However, many objects also rotate around an axis, known as rotational, or angular, motion. Thus, some objects have both translational and rotational motion.

**FACT:** Before proceeding, we need to review some basics about the unit circle. In rotational motion, and depending on the situation, we will use radians (rad), degrees, and revolutions (rev). Recall that a radian measures a distance around an arc ( $s$  = arc length in meters) equal to the length of the arc's radius ( $r$ ). Also recall that  $\pi = \frac{C}{d}$  and rearranging we get  $C = 2\pi r$ .

**FACT:** In discussing angular displacements ( $\Delta\theta$ ), we must transition to describing the translational displacement around an arc ( $\Delta s$ ) in terms of  $\Delta\theta$ . In other words, we need to be able to convert between translational and rotational motion. We will use the following conversions to accomplish this:  $\Delta s = C = 2\pi r = 2\pi \text{ rad} = 1 \text{ rev} = 360^\circ$ . Lastly, recognize that radians are dimensionless and a rad is merely a placeholder to remind you that you're talking about an angle ( $\theta$ ). For example,  $\frac{C}{r} = 2\pi \text{ rad} = 360^\circ$  and when solving for a translational quantity we will cancel out the rad and use the units we are left with, in this case meters (m). Let's do some examples to practice these conversions and turn off radians mode.

**Q17.** Convert  $90^\circ$  to radians; convert 6 radians to degrees; convert 1.5 revolutions to both radians and degrees.

**Q18.** An insect sits at the top of a bike wheel with diameter of 0.75 m. Assuming the wheel turns counterclockwise, what is the angular displacement of the insect before it is squashed under the wheel? What arc length does the beetle travel through before it is squashed?

**FACT:** Rotational kinematics is similar to translational kinematics. You simply need to learn the rotational versions of the kinematic variables and equations (see tables below). The angular velocity ( $\omega$ ; omega) is given in units of radians per second (rad/s) and is equal to the change in theta over the change in time ( $\omega = \frac{\Delta\theta}{\Delta t}$ ). The angular acceleration ( $\alpha$ ; alpha) is given in units of radians per second<sup>2</sup> (rad/s<sup>2</sup>) and is equal to the change in omega over the change in time ( $\alpha = \frac{\Delta\omega}{\Delta t}$ ).

Variable	Translational	Angular	Translational	Rotational
Displacement	$s = r\theta$	$\theta = \frac{s}{r}$	$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
Velocity	$v = r\omega$	$\omega = \frac{v}{r}$	$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
Acceleration	$a = r\alpha$	$\alpha = \frac{a}{r}$	$v^2 = v_0^2 + 2a\Delta x$	$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$
Time	$t$	$t$		

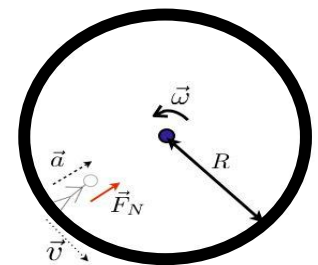
**Q19.** An 18.5 kg bag of sand sits 1.5 m from center of merry-go-round. If merry-go-round rotates at a constant angular speed of 6.58 radians/s. Find the coefficient of static friction between the bag of sand and the merry go round.

**FACT:** The direction of angular vectors is determined by the “right-hand rule”. Wrap the fingers of your right hand in the direction of the rotational displacement, velocity, or acceleration, and your thumb will point in the vector’s direction with up (counterclockwise) being positive. See the diagram to the right.



**Q20.** A car’s tire rotates counterclockwise 3.5 times in 0.75 seconds. What is the angular speed of the tire? Now convert into rpm to ensure you are using the calculator correctly.

**Q21.** A cylindrical space station is rotating counterclockwise at 0.292 rad/s with a 115 meter ladder extending from the middle to the outside. Find the centripetal acceleration at three points extending from the middle, half-way, and the outside ( $r = 115$  m on outside and 0 m in middle).



**Q22.** Answer the following questions using both qualitative and quantitative reasoning. Can an object move in a circle without having centripetal acceleration? Can an object move in a circle without having tangential acceleration?

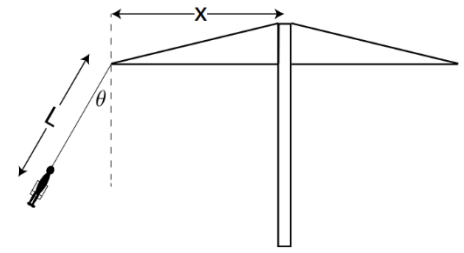
**Q23.** Four children climb on a carousel that initially at rest. The carousel accelerates to 0.4 rad/s within 10 seconds, what is the angular acceleration? What is the linear acceleration (tangential acceleration) 3 meters from the axis of rotation?

**Q24.** Sophia rides a unicycle. If the wheel begins at rest, and accelerates uniformly in a counter-clockwise direction to an angular velocity of 15 rpms in a time 6 sec. find the angular acceleration of the wheel.

**Q25.** Car is traveling at 23 m/s around a flat curve with a radius of curvature of 100 m. The car is going as fast as it can without skidding. What minimum coefficient of friction is necessary for the car to round the curve?

**Q26.** A carpenter cuts a piece of wood with a circular saw. The saw blade accelerates from rest with an angular acceleration of 14 rad/s/s to a max speed of 15,000 rpms. 1). What is the maximum speed of the saw in radians/second? 2). How long does it take to reach this maximum speed? 3). How many complete rotations does the saw make while reaching the maximum speed.

**Q27.** An amusement park ride of radius  $x$  allows children to sit in a spinning swing held by a cable of length  $L$ . At maximum angular speed, the cable makes an angle  $\theta$  with the vertical as shown in the diagram. Derive an expression for the maximum angular speed ( $\omega$ ) of the rider in terms of  $g$ ,  $\theta$ ,  $x$ , and  $L$ .



**FACT:** All objects that have mass attract each other with a gravitational force. The magnitude of that force,  $F_g$ , can be calculated using Newton's Law of Universal

Gravitation:  $F_g = G \frac{m_1 m_2}{r^2}$ . The value for the constant,  $G$ , is  $6.67 \times 10^{-11} \text{ N m}^2/\text{kg}^2$ . Please note that the distance between the masses,  $r$ , in the denominator is the distance between the center of mass of the objects. It is not necessarily the radius of the planet or object. Some textbooks and tutorials replace the  $r$  with  $d$ . For example, if comparing two relatively massive objects like the Earth and Moon, then the  $r$  is the distance between the center of mass of the two objects. But, when calculating the force between a human and Earth, the  $r$  is effectively the radius of Earth, or close enough. As usual, let's first start with some simple problems:

**Q28.** What is the magnitude of the gravitational force of attraction between two asteroids in space, each with a mass of 50,000 kg, separated by a distance of 3800 meters?

**Q29.** Given that the radius of the Earth is about  $6.37 \times 10^6$  meters, determine the mass of the Earth.

**Q30.** The Moon orbits Earth in a circular path at a constant speed. If  $M$  is the mass of Earth,  $m$  is the mass of the Moon, and  $r$  is the Moon's radius, derive an expression for the Moon's orbit speed.

**FACT:** As I have mentioned repeatedly, on the AP Exam you will likely be asked a mathematical reasoning question. When using the Law of Universal Gravitation, note that if either of the masses doubles, then the gravitational force doubles. If the distance between the objects ( $r$ ) is doubled, then the force is quartered. If distance ( $r$ ) increases by three, then the weight decreases by  $3^2 = 9$  fold. Mass is an intrinsic property of an object and does not change.

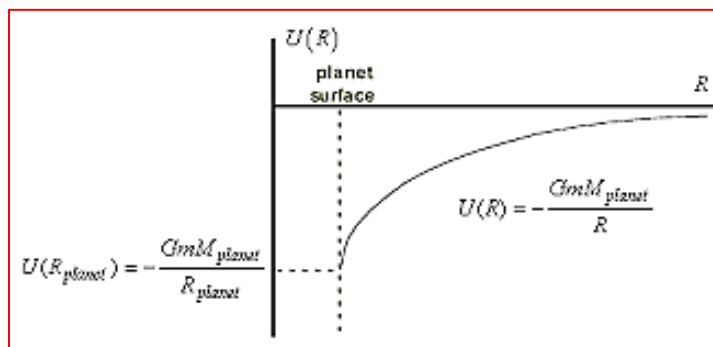
**Q31.** A dwarf planet has  $1/200$  the mass and  $1/5$  the radius of Earth. What is the value for  $g$  (m/s/s) on the surface of the planet?

**Q32.** At the surface of Earth, an object of mass  $m$  has a weight  $w$ . If this object is transported to an altitude that is twice the radius of Earth, what can be said about  $m$  and  $w$ ?

**Q33.** (a) Find the magnitude of the acceleration due to gravity of an object in the troposphere given the mass of the Earth is  $5.97 \times 10^{24}$  kg and the radius of the Earth is 6371 km. Remember that  $G = 6.67 \times 10^{-11}$ . (b) What about a space station 400 km above Earth? How fast does the space station need to move to stay in orbit?

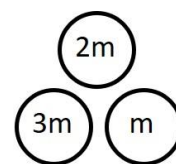
**Q34.** A 2.00 kg object weighs 19.6 N on Earth. If the acceleration due to gravity on Mars is 3.71 m/s/s, what is the object's mass on Mars?

**FACT:** As two objects exert an equal and opposite gravitational force, they cause a displacement. So the gravitational force is doing work ( $W = F \times d$ ) on the objects, which corresponds with the gravitational potential energy. You are familiar with the fact that  $U_g = mgh$  close to the surface of the Earth, but what about objects in deep space? You must use the equation for the law of universal gravitation as the "force" and the displacement as  $r$ . Therefore, this can be expressed as  $U_g = G \frac{m_1 m_2}{r^2} \times r$ , which simplifies to  $U_g = -G \frac{m_1 m_2}{r}$  and again, you may see  $d$  used in place of  $r$ . The negative sign is explained because the zero line is set when the objects are an infinite distance apart. Therefore, any distance less than infinity is negative. Please view the graph below to further understand this fact.



**Q35.** A satellite is currently orbiting Earth in a circular path of radius  $R$  and its kinetic energy  $K$ . If the satellite is moved and enters a new circular orbit of radius  $2R$ , derive an expression for its new kinetic energy.

**FACT:** To calculate the total potential energy in a system, you will need to consider the total number of interacting pairs. Remember that the gravitational potential energy is  $U_g = -G\frac{m_1m_2}{r}$  for each interacting pair. To find the total gravitational potential energy in a system, you will simply add all possible interacting pairs together. Please remember the negative sign. For example, when adding the three equidistant pairs to the right, you would add every combination for the system:  $-G\frac{2m \cdot 3m}{r} - G\frac{3m \cdot m}{r} - G\frac{2m \cdot m}{r} = -G\frac{11m^2}{r}$



**Q36.** A binary star is a celestial phenomenon in which two stars orbit around their mutual center of mass. If the mass of the primary star is  $4m$ , and the mass of the secondary star is  $m$ , what is the gravitational potential energy of the star system if the stars are separated by a distance of  $r$ ?

**FACT:** Gravitational mass indicates how an object responds to a gravitational field. Inertial mass indicates how an object accelerates in response to a net force. These values are equal. You will be asked a question on the AP Exam that will require you to distinguish between these two types of mass. Please view the two Bozeman podcasts for a wonderful explanation with examples.

**Q37.** Design an experiment that would allow you to determine which object, object A or object B, has the largest gravitational mass given they are in the same gravitational field. Next, describe an experiment that would allow you to determine which object, object A or object B, has the largest inertial mass regardless of the gravitational field.

**Q38.** A satellite is orbiting the planet Carlo at a distance  $R$ . An identical satellite also orbits Carlo at a distance  $2R$ . It is a fact that the second satellite at  $2R$  has a smaller tangential speed compared to the first satellite at  $R$ . Derive an expression in terms of  $v$ ,  $m$ ,  $r$ , and fundamental constants to mathematically support this fact.