## Analyzing Simple Harmonic Motion using Circular Motion

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## Objective

We'll use our prior knowledge about circular motion to analyze simple harmonic motion. Our goal is to be able to generate equations for the position, velocity, and acceleration of an object in simple harmonic motion. We will do this by examining the side-to-side motion of an object moving in circular motion.


## Prior knowledge

- I can describe the circular motion of an object using angular quantities such as $\omega$ for angular velocity and $\square$ for angular position.
- I can calculate the linear or tangential velocity of an object moving in uniform circular motion ( $v=\omega r$, $\left.v=\frac{2 \pi r}{T}\right)$
- I can calculate the centripetal acceleration of an object moving in uniform circular motion $a_{c}=\omega^{2} r$
- I can use trigonometry to resolve vectors into perpendicular components
- I understand the relationship between angular displacement, $\square$, and angular velocity, $\omega$ : $\omega=\frac{\Delta \theta}{\Delta t}$

Vocabulary:
Period: the amount of time to complete one cycle of motion
Oscillation: one complete cycle of motion

## Procedure and Questions

1. Observe video 1 on website. What similarities do you notice about the motion of the three objects? What differences do you notice? How are they similar and how are they different? Write down and be prepared to share the properties of the motion of the three objects that appear similar, and the properties of the motions that seems different.
2. Observe video 2 on website. You'll see a pendulum (a brass cylinder on a string), a linear oscillator (red aluminum glider on a low-friction air track with identical springs on either side), and an object moving in uniform circular motion (black cylinder on a rotating platform). Observe the motion of the three objects. In particular, look at the side-to-side motion of the black cylinder compared to the motion of the other two objects.
3. Use the stopwatch to measure the period of each object.
4. Use the the horizontal ruler to find the maximum distance each object moves from the midpoint. In each case, use the mid-point of the object's motion.
5. Next, let's use some features of circular motion and vectors to find an equation for the side-to-side position of the black cylinder. The image below shows the black cylinder on the rotating platform.
As the platform spins, the black cylinder appears to move side to side. Let's call this side-to-side direction the x-axis. It is this side-to-side motion that we'll analyze.

When the platform has rotated through an angle $\theta$, we'll call the apparent $x$-axis position of the black cylinder $\boldsymbol{x}$.


The diagram at right shows a view from above of the rotating platform with the black cylinder on the rim. We'll start at a time from when the black cylinder is at the far right position, and call this position $\theta=0$ and $t=0$.

After some time, $t$, the table will have rotated through an angle $\theta$, as shown. We can use a vector drawing of the position to find the apparent position of the cylinder if viewed from the front of the disk. As shown in the drawing, the position $x$ of the cylinder is:

$$
x=r \cos (\theta)
$$



Since $\omega=\frac{\Delta \theta}{\Delta t}$, at any time, $t$, we know that $\theta=\omega t$. Let's rewrite the apparent position of the cylinder along the x -axis as $x=r \cos (\omega t)$. This position reaches a maximum value when $\omega t$ equals 0 , since $\cos 0=1$. We'll call this maximum position $x_{\text {max }}$. Now we can write the equation for the position of the cylinder:

$$
x=x_{\max } \cos (\omega t)
$$

Let's try this equation out. Using measurements from the video, determine $x_{\max }$ and $\omega$ for the motion of the black cylinder. Remember to use the middle of the of the cylinder to measure the position. Write the equation for the horizontal position of the black cylinder as a function of time:
5. Let's explore whether this equation actually works. Will it accurately predict the location of the black cylinder at any time? Let's agree on a starting point for our motion. We'll say $t=0 \mathrm{~s}$ at frame 252, because that is the first time the cylinder is nearest the maximum positive position, $x_{\max }$. Notice that we can't find the exact time when the cylinder is at the maximum position; there is at least one frame of uncertainty because we can't be sure that the cylinder is exactly at its maximum position at any of the video frames. Set the stopwatch to zero at this time. Use your equation to predict the position of the cylinder at $t=5.533 \mathrm{~s}$, for example. Advance the video to this time and check whether your prediction is correct. Again, note that there is at least 1 frame of uncertainty
6. To verify that this process works for the glider and pendulum as well, let's repeat this process. Determine the equation for the position of the glider. Use your equation to make a prediction about the position of the glider at some arbitrary time. Determine whether your prediction is valid or not.
7. Now we'll look at the velocity of the black cylinder. Specifically, we'd like to find the $x$-axis velocity in the same way we found the side-to-side position in the previous section. Begin by using measurements from the video to determine the tangential velocity of the black cylinder.
8. Looking at the green vectors in the drawing at right, we can see that the velocity parallel to the $x$-axis is $v_{x}=-\omega r \sin (\theta)$. As before, we know that $\theta=\omega t$, so we'll rewrite the apparent position of the cylinder as $v_{x}=-\omega r \sin (\omega t)$. Again, $r$ is equal to the maximum position $x_{\text {max }}$. Now we can write the equation for the velocity of the cylinder along the x -axis:

$$
v_{x}=-x_{\max } \omega \sin (\omega t)
$$


9. While we can't easily measure the velocity of the cylinder, we can still use this equation to make predictions we can test using the video. For example, let's find a time (other than $t=0$ ) when the cylinder's
velocity along the x -axis is zero. To find a time when the the velocity is zero, solve the velocity equation, setting $v_{x}$ equal to 0 . Remember that $\omega=\frac{2 \pi}{T}$. You'll find that there are many possible solutions. Select any solution (except $t=0 \mathrm{~s}$ ) and test your prediction.
10. Now we can move to our last equation: the $x$-direction acceleration for the cylinder. We use the blue vectors as shown at right to find the x component of the centripetal acceleration. The centripetal acceleration is $a_{c}=\omega^{2} r$. Using the blue vectors, we see that the component of $\mathbf{a}_{c}$ that is parallel to the xaxis is: $a_{x}=-\omega^{2} r \cos (\theta)$ Once again, making our substitution for $\theta=\omega t$, we can rewrite the apparent position of the cylinder as $a_{x}=-\omega r \sin (\omega t)$. Again, $r$ is equal to the maximum position $x_{\text {max. }}$. Now we can write the equation for the velocity of the cylinder along the $x$-axis:

$$
a_{x}=x_{\max } \omega^{2} \cos (\omega t)
$$


11. Use this equation to predict a time when the acceleration of the cylinder is at a minimum value.

## Summary Questions

Write down the three equations for position, velocity, and acceleration for objects in simple harmonic motion. Use the three equations to complete the table for the motion of the red glider during the first complete oscillation:

| frame | time | position | velocity | acceleration |
| :--- | :--- | :--- | :--- | :--- |
| 112 | 0 | maximum right | 0 |  |
|  |  |  | maximum leftward |  |
|  |  |  |  | maximum rightward |

