AP Physics 1: How to Derive Expressions on the AP Physics Exam

College Board will often ask you to drive expressions on the free response section of the exam. From College Board: "Derive is more specific and indicates that the students need to begin their solutions with one or more fundamental equations, such as those given on the AP Physics Exam equation sheet. The final answer, usually algebraic, is then obtained through the appropriate use of mathematics." You will be asked to derive expressions on every exam you take in this course to prepare you for the AP Exam. This is your guide to derivations and my three step plan to successfully derive an expression to earn maximum credit.

Step 1: Translate the physics to identify exactly what is being asked in the derivation and come up with a general plan. Do not worry about specific numbers; most variables will be given to you in relative terms. For example, if a planet has a mass of *M* and you are told a second planet is four times more massive, than planet two would be expressed as *4M*. Another problem may state an object is at position *D* at time t_D. So you would substitute *D* for change in position (Δx) in the UAM equations and t_D for time in the UAM equations (see step 3 for details).

Step 2: Start with equations and formulas from the AP Physics equations sheet and any other fundamental constants. For example, you might start with $V_{fy} = V_{0y} + a\Delta t$ for a projectile, where "a" would be replaced with the fundamental constant, "g", for the acceleration due to gravity.

Step 3: Replace variables in the equation with the relative terms you derived in step 1 and simplify algebraically. Referring to the UAM equation in step 2, if you were told the time was reduced by half and the initial launch velocity tripled, than you could write: $V_{fy} = 3V_{0y} + g\frac{t}{2}$. Referring to the variables in step 1, you might write: $D = V_{0t} + 1/2at_D^2$. Clearly show all steps of your derivation to receive maximum credit. Let's practice deriving expressions:

Unit 1.1: Two balls are launched off the edge a cliff of height *h* with an initial velocity v_0. The red ball is launched horizontally. The green ball is launched at an angle of θ above the horizontal. Neglect air resistance.

(a). Derive an expression for the time the red ball is in the air.

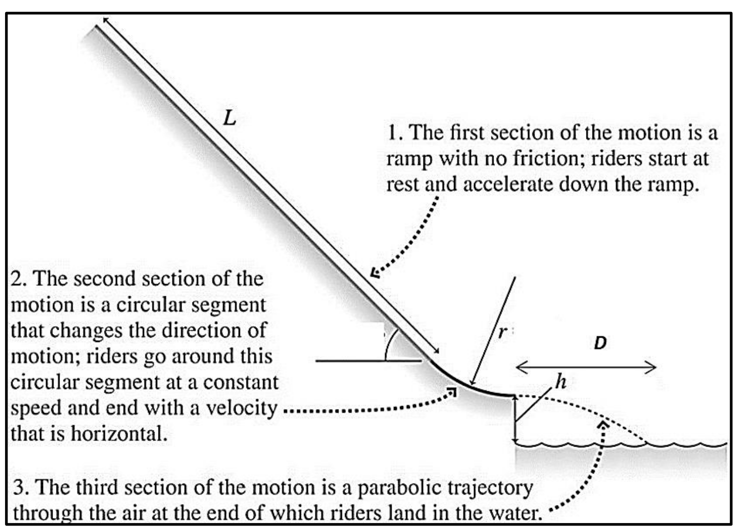
(b). Derive an expression for the horizontal distance traveled by the red ball while it is in the air.

(c). Derive an expression for the time the green ball is in the air. You will end with a quadratic equation, which you will not be expected to solve on the AP Physics 1exam. However, deriving more complicated expressions will further familiarize you with the derivation process and help you to earn maximum points on exams.

(d). Derive an expression for the horizontal distance traveled by the green while it is in the air. You will need your derived expression from (c) in your answer.

Unit 1.2: A slide of length *L* makes an angle θ above the horizontal. The end of the slide with a radius (*r*, not used in this unit) launches Sophia from a height (*h*) into the water with a horizontal velocity of V_x. Sophia flies through the air for time, t_p, and gently lands a distance, *D*, from the end of the frictionless slide.

(a). Derive an expression for Sophia's horizontal launch velocity (V_x) in terms of *L*, *h*, θ, t_p, and any other fundamental constants. Note: You may not need to use all of the terms.



(b). Mr. Pedersen wants to design a new slide that will double the horizontal distance, *D*, Sophia will launch before gently splashing into the water. He plans to accomplish this by changing the length (*L*) of the slide, but leaving the angle (θ) and height (*h*) constant. Using your expression from part (a), derive a new expression in terms of *L, h*, θ, *D*, t_p, and any other fundamental constants that will result in a new horizontal range, *2D*. Note: You may not need to use all of the terms.

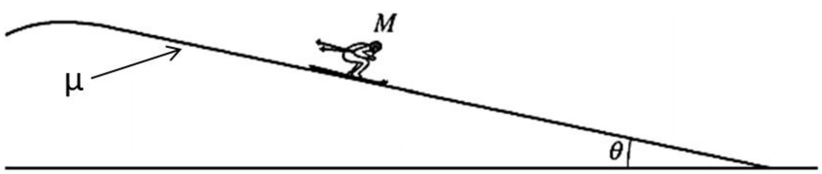
(c). Mr. Pedersen runs out of materials to increase the length, *L*, of the slide, so he is unable to complete the project and must devise a new plan with the same end goal. He must leave the length of the slide and the angle of the slide the same. Derive a new expression in terms of *L, h*, θ, *D*, t_p, and any other fundamental constants that will result in a new horizontal range, *2D*. Note: You may not need to use all of the terms.

Unit 2.1: A bullet of mass *M* is fired from a gun and leaves the barrel with a velocity of V_b. The barrel of the gun is of length L_g. The net force (F_b) acting on the bullet in the barrel is constant, which causes a constant acceleration of a_b.

(a). Derive an expression for the acceleration of the bullet (a_b) in terms of the velocity of the bullet (V_b), length of the gun barrel (L_g), and any other fundamental constants.

(b). Using your expression in part (a), derive an expression for the net force (F_b) acting on the bullet while in the barrel. Express your answer in terms of the velocity of the bullet (V_b), length of the gun barrel (L_g), mass of the bullet (*M*), and any other fundamental constants.

Unit 2.2: A skier of mass *M* is skiing down a hill that makes an angle θ with the horizontal, as shown in the diagram. This particular slope is a bit rough and has a small coefficient of friction, μ. The skier starts from rest at t = 0 and is subject to a constant frictional force (*F*) as she descends the hill. Neglect air resistance.

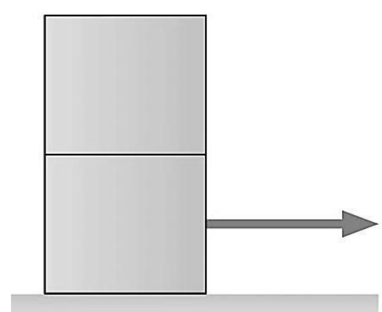


(a). Derive an expression for the friction force (*F*) that is exerted on the skier. Express your answer in terms of *M*, θ, μ, and fundamental constants.

(b). Derive an expression for the acceleration of the skier (*a*) down the incline. Express your answer in terms of *M*, θ, μ, and fundamental constants.

(c). The skier decides to go down the hill again, but in another location with a new coefficient of friction, μ_2. This time she skies down the slope with a constant velocity. Using your derivations from parts (a) and (b), derive an expression for the new coefficient of friction, μ_2. Express your answer in terms of *M*, θ, and fundamental constants.

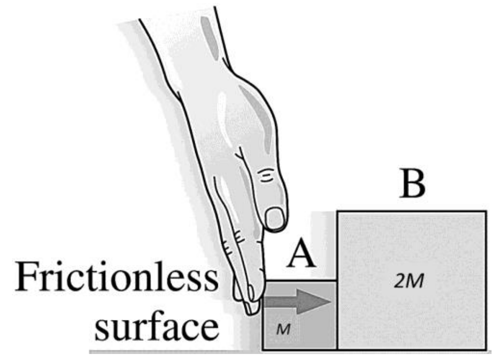
Unit 2.3: Two identical crates each with mass, *M*, are stacked as shown in the figure. The bottom block is free to slide on a frictionless surface. The coefficient of static friction between the two blocks is μ_{sf}. A force (*F*) is applied to the two-crate system.



(a). Derive an expression for the net horizontal force (*F*) of the two-crate system. Your expression should be in the terms *F*, *M*, *a*, and any other fundamental constants.

(b). Derive an expression for the maximum horizontal force (F_{max}) that can be applied to the lower crate without the upper crate slipping. Your expression should be in the terms μ_{sf}, *F* , *M*, and any other fundamental constants.

Unit 2.4: Block A has a mass *M* and is being pushed by a hand with a force, F_H. In front of block A is another object, block B, with a mass *2M*. The two blocks move together. Derive an expression for the force that block A exerts on block B (F_{AB}) as they move to the right with a constant acceleration (a_x). Your final expression should be in terms of *M*, F_H, F_{AB}, and other fundamental constants.



Unit 2.5: Orthopedic surgeons will often use traction devices to help prevent bones from fusing incorrectly after a serious injury. Several magnet students are challenged to use their expansive knowledge of physics to design a traction device using rope, a frictionless pulley, and hanging masses. The rope must make the same angle θ on both sides of the pulley so that the net force of the rope on the pulley is horizontal and to the right (as shown below). The angles can be adjusted to control the amount of traction. The students believe it would be best to derive some expressions that can be used for determining traction.

(a). Derive an expression for the force of the leg on the pulley (F_{LP}) in terms of the hanging mass *M*, angle θ, and any other fundamental constants.

(b). Due to a budget constraint, the students only have one hanging mass (*M*), but in order to design an effective traction device, they need a method to readily change the force of the pulley on the leg. Using the expression from part (a), derive an expression that could be used to determine the angle required for any desired force of the pulley on the leg. Your expression should be in terms of the force of the leg on the pulley (F_{LP}) the hanging mass (*M*), angle θ, and any other

Link to article: "Students' Understanding Physics Concept of Traction Therapy"